CONTINUITY AND DIFFERENTIABILITY

CASE/SOURCE BASED QUESTIONS

Read the passage given below and answer the following questions:

QNO. 1 Let $x = f(t) and\ y = g(t)$ be parametric forms with t as a parameter, then $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \frac{g'(t)}{f'(t)} where\ f'(t) \neq 0$. On the basis of above information , answer the following questions:

1.1 The derivative of $f(tanx) w.r.t\ g(secx)$ at $x = \frac{\pi}{4}$, where $f'(1) = 2\ and\ g'(\sqrt{2}) = 4$ is :

(A) $\frac{1}{\sqrt{2}}$

(B) $\sqrt{2}$

(C) 1

(D) 0

1.2 The derivative of $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ w.r.t $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ is:

(A) -1

(B) 1

(C) 2

(D) 4

1.3 The derivative of e^{x^3} w.r.t $\log x$ is:

(A) e^{x^3}

(B) $3x^2e^{x^3}$

(C) $3x^3e^{x^3}$

(D) $3x^3e^{x^3} + 3x$

1.4 The derivative of $\cos^{-1}(2x^2 - 1)$ w.r.t $\cos^{-1} x$ is :

(A) 2

(B) $\frac{-1}{2\sqrt{1-x^2}}$

(C) $\frac{2}{x}$

(D) $1 - x^2$

1.5 If $y = \frac{1}{4}x^4\ and\ x = \frac{2}{3}x^3 + 5$ then $\frac{dy}{dx}$ is equal to :

(A) $\frac{2}{27}x^2(2x^3+15)^3$

(B) $\frac{2}{7}x^2(2x^3+15)^3$

(C) $\frac{2}{27}x(2x^3+5)^3$

(D) $\frac{2}{7}(2x^3+15)^3$

Q.NO. 2 The derivative of $f\ at\ x=c$ is defined by : $f(x)=\lim_{h\to 0}\frac{f(c+h)-f(c)}{h}$

A function is said to be differentiate at point c if left hand derivative at x=c is equal to the right hand derivative at x=c . Similarly a function is said to be differentiable in an interval (a,b) if it is not differentiable at every point (a,b). Based on the above information , answer the following questions.

2.1 Derivative of $f(x)=\cos\sqrt{x}$ is:

(A) $-\sin(\sqrt{x})$

(B) $\frac{-\sin(\sqrt{x})}{2\sqrt{x}}$

(C) $\sin(\sqrt{x})$

(D) $\frac{1}{2}\sin(\sqrt{x})$

2.2 If $y=asint, x=acost$, then $\frac{dy}{dx}$ is :

(A) $\cos t$

(B) $-\tan t$

(C) $-\cot t$

(D) $\sin t$

2.3 $f(x)=|x|$ is:

(A) Differentiable at all points x$\in R$

(B) Differentiable at all points x$\in R-\{0\}$

(C) Not differentiable at x=1

(D) None of these.

2.4 Derivative of the function $f(x)=\sin(x^2)$ is:

(A) $2\cos(x^2)$

(B) $2\text{x}\cos(x^2)$

(C) $2\text{x}^2\cos(x^2)$

(D) $2\cos(x)$

2.5 If $y+\sin y=\cos x$, then $\frac{dy}{dx}$ is :

(A) $\frac{-\sin x}{1+\cos y}$

(B) $\frac{\cos x}{1+\sin y}$

(C) $\frac{\cos y}{1+\sin x}$

(D) $\frac{-\cos x}{1+\sin y}$

Q. NO . 3: Logarithmic differentiation is a powerful technique to differentiate functions of the form f(x)=$u(x)^{v(x)}$ where $u(x), v(x)$are differentiable functions and f and u need to be positive functions.

Let function y=f(x)=$u(x)^{v(x)}$ then y'=$\frac{v(x)}{u(x)}u'$(x) +v'(x).log[u(x)]

On the basis of above information, answer the following questions.

(3.1)Differentiate x^x w.r.t x

(A) $x^x(1+logx)$ (B) $x^x(1-logx)$ (C) $-x^x(1+logx)$ (D)$x^x(logx)$

(3.2)Differentiate $x^x + a^x + x^a + a^a \ w.r.t\ x$

(A) $x^x(1+logx)$ (B) $a^x loga + ax^{a-1}$ (C) $x^x(1+logx)$ +$a^x loga$ (D)$x^x(1+logx)$ + $a^x loga +$ ax^{a-1}

(3.3) If x= $e^{\frac{x}{y}}$ then dy /dx

(A) $\frac{x-y}{logx}$ (B) $\frac{xy}{xlogx}$ (C) $\frac{x+y}{xlogx}$ (D)$\frac{x-y}{xlogx}$

(3.4) If y=$(2-x)^3(3+2x)^5$ then find dy/dx

(A)$(2-x)^3(3+2x)^5[\frac{15}{3+2X}-\frac{8}{2-x}]$ (B)$(2-x)^3(3+2x)^5[\frac{15}{3+2X}+\frac{8}{2-x}](C)(2-x)^3(3+2x)^5[\frac{10}{3+2X}-\frac{3}{2-x}]$
(D)$(2-x)^3(3+2x)^5[\frac{10}{3+2X}+\frac{3}{2-x}]$

(3.5) If y= $x^x e^{2x+5}$ then dy/dx

(A) $e^{2x+5}(3+logx)$ (B) $x^x(3+logx)$ (C) $x^x e^{2x+5}$ (D) $x^x e^{2x+5}(3+logx)$

Q. NO.4 : Derivative of y=f(x) w.r.t x(if exists) is denoted by $\frac{dy}{dx}$ is called the first order derivative of y. If we take derivative of $\frac{dy}{dx}$again , then we get $\frac{d}{dx}(\frac{dy}{dx})$
=$\frac{d^2y}{dx^2}$ $is\ called\ the\ second\ order\ derivative\ of\ y. Simi$larly,$\frac{d}{dx}(\frac{d^2y}{dx^2})$ is known as third order derivative and so on.

Based on the above information, answer the following questions.

4.1 . If y=$\tan^{-1} e^x \frac{\log(\frac{e}{x^2})}{\log(ex^2)}$ + $\tan^{-1}(\frac{3+2logx}{1-6logx})$ then $\frac{d^2y}{dx^2}$ is equal to

(A) 2 (B) 1 (C)0 (D)-1

4.2. If u= x^2 +y^2 and x= s+3t , y=2s-t then $\frac{d^2y}{dx^2}$ is equal to

(A) 12 (B) 32 (C) 36 (D) 10

(4.3) If f(x)=2logsinx then $\frac{d^3y}{dx^3}$ is equal to

A)$-2\,cosec^3$x (B) -2x $cosec^2$x (C))2 $cosec^2$x (D)$-2\,cosec^2$x

(4.4) If f(x)=e^xsinx then $\frac{d^3y}{dx^3}$ is equal to

(A) $2\ e^x(cosx + sinx)$ (B) $2\ e^x(cosx - sinx)$ (C) $2\ e^x(sinx - cosx)$ (D) $e^x(cosx - sinx)$

(4.5) If y^2 =ax^2+bx+c then $\frac{d}{dx}$(y^3y_2)=

(A) 1 (B) -1 (C) $\frac{4ac-b^2}{a^2}$ (D)0

Q.NO. 5 Read the passage given below and answer the following questions.

Mr. Mohan is a mathematics teacher of Kendriya Vidyalaya teaching method of logarithmic differentiation his students with the help of a flow -chart. Method of logarithmic differentiation says that if functions of the form y = $[f(x)]^{g(x)}$ then

Y′ = $[f(x)]^{g(x)}\left\{\frac{g(x)}{f(x)} \cdot \frac{d}{dx}\ f(x) + \log f(x).\frac{d}{dx}\ g(x)\right\}$

Let f(x) = sin x and g (x) = log x

(A)f’ (x) is equal to

(a) cos x

(b) - cos x

(c) sin x

(d) - sin x

(B) g′ (x) is equal to

(a) 1/x

(b)x

(c) - 1/x

(d) - x

(C) $\frac{d}{dx}$ (log sin x) is equal to

(a) 1/sinx

(b) 1/cos x

(c) tan x

(d) cot x

(D) $\frac{dy}{dx}$ is equal to

(a) $(\sin x)^{\log x}\left\{\frac{\log\sin x}{x} + \cot x\,.\log x\right\}$

(b) $(\sin x)^{\log x}\left\{\frac{\log\sin x}{x} + \tan x\,.\log x\right\}$

(c) $(\sin x)^{\log x}\left\{\frac{\log\sin x}{x} + \cos x\,.\log x\right\}$

(d) $(\sin x)^{\log x}\left\{\frac{\log\sin x}{x} + \sin x\,.\log x\right\}$

Q.NO. 6 Read the passage given below and answer the following questions.

Mr. Mohan is a mathematics teacher ofKendriya Vidyalaya teaching method of higher order derivatives of his students .$\frac{dy}{dx}$is first order derivative of y with respect to x and the derivative of

$\frac{dy}{dx}$ with respect to x as the second order derivative of y with respect to x will be d^2y/dx^2 . The n^{th} order of derivative of y with respect to x will be denoted by $d^n y/dx^n$.

Let y = tan x + sec x

(A) $\frac{d}{dx}$ $(\tan x)$ is equal to

(a) Sec^2 x
(b) sec x. tan x
(c) sec x
(d) – sec x

(B)$\frac{d}{dx}$ $(\sec\ x)$ is equal to

(a)Sec^2 x
(b)sec x. tan x
(c)sec x
(d)sec x

(C)$\frac{dy}{dx}$ is equal to

(a) 1/(1 – sin x)
(b) 1/(1+cos x)
(c) 1/(sin x + cos x)
(d) 1/(sin x – cos x)

(D) d^2y/dx^2 is equal to

(a) cos x/$(1 + \sin x)^2$
(b) cos x/$(1 - \sin x)^2$
(c) sin x/$(1 + \cos x)^2$
(d) sin x/$(1 - \cos x)^2$

Q . NO. 7

Parametric equation of the path followed by a projectile is in the form of a parabola is $x = at^2, y = 2at$where 't' is the parameter. Based on the above information answer the following:

A. The equation of the projectile is
   (i) y^2=-4ax
   (ii) x^2=-4ay
   (iii) y^2=4ax
   (iv) x=-4ay
B. Find $\frac{dy}{dx}$ $in\ term\ of\ 't'$
   (i) t
   (ii) $\frac{1}{t}$
   (iii) $\frac{2a}{t}$
   (iv) $\frac{1}{at}$
C. Find $\frac{dy}{dx}$ $in\ term\ of\ 'y'$
   (i) $\frac{2a}{y}$

(ii) $\frac{y}{2a}$

(iii) $\sqrt{\frac{a}{y}}$

(iv) $\frac{a}{2y}$

D. Find $\frac{d^2y}{dx^2}$

(i) $\frac{-1}{t^3}$

(ii) $\frac{-a}{t^3}$

(iii) $\frac{-1}{2at^3}$

(iv) $\frac{-a}{2t^3}$

E. Find derivative of $\frac{d^2y}{dx^2}$ (considering it as a function) with respect to $\frac{dy}{dx}$

(i) $\frac{-2}{3t^2}$

(ii) $\frac{-3}{2at^2}$

(iii) $\frac{-2a}{3t^2}$

(iv) $\frac{-3a}{2t^2}$

Q . NO. 8

A particle is moving on a path given by the function S(t) = $t^2 - 6t + 18$, where t is the time elapsed and S m is the distance covered by the particle at a particular time t sec.

Based on the above mentioned facts answer the following:

A. Find the velocity of the particle

(i) 2t-5 m/sec

(ii) 2t-6 m/sec

(iii) 2 cm/sec

(iv) 2t-6 cm/sec

B. Find the acceleration of particle after 5 secs

(i) $2m^2/sec$

(ii) 10 m/sec^2

(iii) $5cm^2/sec$

(iv) 2 m/sec^2

C. Find the maximum height covered by the particle

(i) 18 m

(ii) 13 cm

(iii) 9 m

(iv) 9 cm

D. Find the time taken by the particle to reach maximum height

(i) 3sec

(ii) 6sec

(iii) 2sec

(iv) 18sec

E. Write down acceleration in term of velocity of the particle

(i) $\frac{ds}{dt}$

(ii) $\frac{d^2s}{dt^2}$

(iii) $\frac{dv}{dt}$

(iv) $\frac{d^2v}{dt^2}$

Q.NO. 9. Mrs. Rekha of model school is teaching chain rule to her students with the help of a flow-chart. The chain rule says that if h and g are functions and f(x) = g(h(x)), then

$f'(x)$ = $(g(h(x))'$ = $g'\left(h(x)\right)h'$ (x)

Let f(x) =sin x and g(x) = x^3

A) fog(x) = ...

a) sin x^3 b) sin^3x c) sin 3x d) 3 sin x

B) gof(x) =..........

a) sin x^3 b) sin^3 x c) sin 3x d) 3 sin x

C) $\frac{d}{dx}$(sin^3x) = ...

a) $cos^3\ x$ b) 3 sin x cos x c) 3 $sin^2x\cos x$ d) - cos^3x

D) $\frac{d}{dx}$sinx^3 = ...

a) cos(x^3) b) – cos (x^3) c) $3x^2\sin(x^3)$ d) $3x^2\cos(x^3)$

E) $\frac{d}{dx}$(sin 2x) at x = $\frac{\pi}{2}$ is ....

a) 0 b) 1 c) 2 d) – 2

Q.NO.10. Let x = f(t) and y = g(t) be parametric forms with t as a parameter, then $\frac{dy}{dx} = \frac{dy}{dt} x\ \frac{dt}{dx}$ = $\frac{g'(t)}{f'(t)}$ where $f'(t) \neq 0$.

On the basis of above information, answer the following questions:

A) The derivative of f(cot x) w.r.t g (cosec x) at x = $\frac{\pi}{4}$ where $f'(1)$ = 2 and $g'(2)$ = 4 is ....

a) $\frac{1}{\sqrt{2}}$ b) $\sqrt{2}$ c) 1 d) 0

B) The derivative of $\sin^{-1}x$ wrt $\cos^{-1}x$ is ....

a) – 1 b) 1 c) 2 d) 4

C) The derivative of log x w.r.t e^{x^3} is ....

a) $\frac{1}{e^{x^3}}$ b) $\frac{1}{3x^2.2e^{x^3}}$ c) $\frac{1}{3x^3e^{x^3}}$ d) $\frac{1}{3x^2e^{x^3}+3x}$

D) The derivative of $\cos^{-1}(2x^2-1)$ wrt $\cos^{-1}x$ is ....

a) 2 b) $\frac{-1}{2\sqrt{1-x^2}}$ c) $\frac{2}{x}$ d) 1 - x^2

E) If y = $\frac{1}{4}r^4$ and r = $\frac{2}{3}x^3 + 5$, then $\frac{dy}{dx}$ = ……………

a) $\frac{2}{27}x^2(2x^3+15)^3$ b) $\frac{2}{7}x^2(2x^3+15)^3$ c) $\frac{2}{27}\ x\ (\ 2x^3+5)^3$

d) $\frac{2}{7}(2x^3+15\ )^3$

Q No. Read the passage given below and answer the following questions……..

11. $1.\ Let\ f(x)\ be\ a\ differentiable\ function\ of\ x\ whose\ second\ order$ $derivative\ is\ exists. we\ denote\ the\ second\ order\ derivative\ of$

$$y\ with\ respect\ to\ x\ by\ \frac{d^2y}{dx^2}\ or\ y_2.$$

$based\ on\ the\ given\ information\ answer\ the\ following\ questions:-$

(i) $If\ y = 3e^{2x} + 2e^{3x}, then\ the\ value\ of\ \frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y, is$

$(a)\ 0$ $(b)\ 2$ $(c)\ 3$ $(d)\ 1$

(ii) $if\ x = a(\theta + \sin\theta)\ and\ y = a(1-\cos\theta), then\ the\ value\ of$ $\frac{d^2y}{dx^2}\ at\ \theta = \frac{\pi}{4}$

$(a)\frac{1}{a}(b)\ a$ $(c) - a$ $(d) - \frac{1}{a}$

(iii) $If\ x = (2\cos t - \cos 2t)\ and\ y = (2\sin t - \sin 2t), then\ the$ $value\ of\ \frac{d^2y}{dx^2}\ at\ t = \frac{\pi}{2}$

$(a)\frac{2}{3}(b) - \frac{2}{3}(c) - \frac{3}{2}(d)\frac{3}{2}$

(iv) $If\ y = e^{-x}\cos x, then\ the\ value\ of\ \frac{d^2y}{dx^2}\ is,$

$(a)\ 2e^{-x}\cos x\ (b)e^{-x}\cos x$

$(c)e^{-x}\sin x\ (d)\ \ 2e^{-x}\sin x$

(v) $If\ y = A\ cos\ nx + Bsin\ nx, then\ the\ value\ of\ \frac{d^2y}{dx^2}\ is,$

$(a)n^2y$ $(b) - n^2y$

$(c)\ ny$ $(d) - ny$

Q.NO. 12. $If\ the\ derivative\ of\ e^x\ w.r.t.\ x\ =\ \frac{d}{dx}(e^x) =\ e^x$

$The\ derivativeof\ \log x\ w.r.t.\ x\ =\ \frac{d}{dx}(\log x) = \frac{1}{x}$

$The\ derivative\ of\ a\ function\ that\ is\ the\ power\ of\ another$

$function\ i.e.\ y\ =\ f(x) = [u(x)]^{v(x)}, then$

$\therefore \frac{dy}{dx} = y[\frac{v(x).u'(x)}{u(x)} + v'(x).log\ u(x)]$

(i) $If\ y = x^x, then\ the\ value\ of\ \frac{dy}{dx}, is$

$(a)\ y(1+\log x)(b)\ y\log x$

$(c)\ x.x^{x-1}(d)\ 1+\log x$

(ii) $If\ x^y = e^{x-y}, then\ the\ value\ of\ \frac{dy}{dx}, is$

$(a)\frac{\log\ x}{(1+\log x)^2}(b) - \frac{\log\ x}{(1+\log x)^2}$

$(c) - \frac{(1+\log x)}{\log x}(d)\frac{\log x}{(1-\log x)^2}$

(iii) $If\ x^y = y^x, then\ the\ value\ of\ \frac{dy}{dx}$

$(a)\frac{y(y-x\log y)}{x(x-y\log x)}(b)\frac{x(x-y\ log\ x)}{y(y-x\log y)}$

$(c)\frac{y(y+x\log y)}{x(x+y\log x)}(d)\frac{(x-y\ log\ x)}{(y-x\log y)}$

(iv) $If\ xy = e^{x-y}, then\ the\ value\ of\ \frac{dy}{dx}$

$(a)\frac{y(x-1)}{x(y+1)}(b)\frac{y}{x}$

$(c)\frac{yx-1)}{xy-1)}(d)\frac{y-1}{x-1}$

(v) $If\ x = e^{x/y}, then\ find\ the\ value\ of\ \frac{dy}{dx}$

$(a)\frac{y-x}{\log x}(b)\frac{\log x}{x-y}$

$(c)\frac{x-y}{\text{x}\log x}(d)\frac{y+x}{\log x}$

Q.NO. 13 If a relation between x and y is such that y cannot be expressed in terms of x, then y is called an implicit function of x. When a given relation expresses y as an implicit function of x and we want to find $\frac{dy}{dx}$, then we differentiate every term of the given relation with respect to x, remembering that a term in y is first differentiated with respect to y and then multiplied by $\frac{dy}{dx}$.

Based on the above information, find the values of $\frac{dy}{dx}$ in each of the following questions.

(i) $x^3 + x^2y + xy^2 + y^3$

(a) $\frac{(3x + 2xy + y)}{x^2 + 2xy + 3y^2}$

(b) $\frac{-(3x^2 + 2xy + y^2)}{x^2 + 2xy + 3y^2}$

(c) $\frac{(3x^2 + 2xy + y^2)}{x^2 - 2xy + 3y^2}$

(d) $\frac{(3x^2 + x + y^2)}{x^2 + xy + 3y^2}$

(ii) $x^y =$

(a) $\frac{x - y}{1 - \log_e x}$

(b) $\frac{x + y}{1 - \log_e x}$

(c) $\frac{x - y}{x(1 + \log_e x)}$

(d) $\frac{x + y}{x(1 - \log_e x)}$

(iii) $e^{\sin y} = xy$

(a) $\frac{-y}{x(y\cos y - 1)}$

(b) $\frac{y}{y\cos y - 1}$

(c) $\frac{y}{y\cos y + 1}$

(d) $\frac{y}{x(y\cos y - 1)}$

(iv) $\sin^2 x + \cos^2 y = 1$

(a) $\frac{\sin 2y}{\sin 2x}$

(b) $-\frac{\sin 2x}{\sin 2y}$

(c) $-\frac{\sin 2y}{\sin 2x}$

(d) $\frac{\sin 2x}{\sin 2y}$

Q.NO. 14 Second order derivative is the derivative of a function. In physics, score quantities are also represented through second derivative e.g. acceleration.

If the speed(s) of an object is changing with time(t), then its acceleration is represented by $\frac{d^2s}{dt^2}$.

Based on the above information, answer the following questions.

(i) If $y = 15x + \frac{15}{x^2}$, then $\frac{d^2y}{dx^2}$ is

(a) $\frac{30}{x^3}$

(b) $\frac{-30}{x^3}$

(c) $\frac{90}{x^4}$

(d) $\frac{-90}{x^4}$

(ii) If $y = a + bx^2$, then $\frac{d^2y}{dx^2}$ is

(a) $\frac{dy}{dx}$

(b) $\frac{1}{x}\frac{dy}{dx}$

(c) $\frac{2}{x}\frac{dy}{dx}$

(d) 2bx

(iii) If $y = \cos(x^3)$, then $\frac{d^2y}{dx^2}$ is

(a) $-9x^4 \cos x^3 - 6x \sin x^3$

(b) $9x^4 \cos x^3 + 6x \sin x^3$

(c) $-9x^4 \cos x^3 + 6x \sin x^3$

(d) $9x^4 \cos x^3 - 6x \sin x^3$

(iv) If $y = \sin(\log x)$, then $\frac{d^2y}{dx^2}$ is

(a) $\frac{1}{x^2}[\cos(\log x) - \sin(\log x)]$

(b) $\frac{1}{x^2}\sin(\log x) - \cos(\log x)$

(c) $\frac{1}{x^2}[\cos(\log x) + \sin(\log x)]$

(d) $-\frac{1}{x^2}[\sin(\log x) + \cos(\log x)]$

Q.NO.15 A potter made a mud vessel, where the shape of the pot is based of *f(x)*= | *x*-3|+| *x*-2|, where *f(x)*represents the height of the pot.

Answer the following questions given below.

1 When x>4 what will be the height in term of x?

a. x-2
b. x-3
c. 2x-5
d. 5-2x

2 Will the slope vary with x value ?

a. Yes
b. No
c. may or may not vary
d. none of these

3 What is $\frac{dy}{dx}$ at x=3

a. 2
b. -2
c. Function is not Differentiable
d. 1

4 When the x value lies between (2,3) then the function is

a. 2x-5
b. 5-2x
c. 1
d. 5

5 If the potter is trying to make a pot using the function *f(x)*=[x], will he get pot or not ? why ?

a. Yes, because it is a continuous function
b. Yes, because it is not continuous
c. No, because it is a continuous function
d. No, because it is not continuous

Q.NO.16 A function is continuous of x=c if the function is defiened at x=c and if the value of the function at x=c equals the limit of the function at x=c i.e $\lim_{X \to C} f(x)$ f(x)=f(c)
if f is not continuous at c, we say f is discontinuous at c and c is called a point of discontinuity of f. Based on the above information answer the following questions:-

1 The number of points of discontinuity of *f(x)*=[x] in [3,7] is

a. 4
b. 5
c. 6
d. 8

2 Suppose f anf g be two real functions continuous at a real real number c then

a. f+g is continuous at x=c
b. f+g is discontinuous at x=c
c. f+g may or may not continuous at x=c
d. none of above

3 The value of k so that the given function *f(x)* is continuous at x=5 $f(x) = \begin{cases} k\,n+1 \ , & x \le 5 \\ 3x-5 \ , & x \ge 5 \end{cases}$

a. 9/5

b. 5/9
c. 1/9
d. 1/5

4 The value of k so that given function *f(x)* is continuous at x=2 $f(x) = \begin{cases} kx^2, & x \leq 2 \\ 3, & x \geq 2 \end{cases}$

a. 1
b. $\frac{1}{4}$
c. $\frac{3}{4}$
d. $\frac{11}{4}$

Q.NO.17 Read the passage given below and answer the following questions……..

Logarithmic Differentiation.

Assume that the function has the form $y = f(x)^{g(x)}$ where both f and g are non-constant functions. Although this function is not implicit, it does not fall under any of the forms for which we developed differentiation formulas so far. This is because of the following.

- In order to use the power rule, the exponent needs to be constant.
- In order to use the exponential function differentiation formula, the base needs to be constant.

Thus, no differentiation rule covers the case $y = f(x)^{g(x)}$

- These functions sill can be differentiated by using the method known as the logarithmic differentiation.

To differentiate a function of the form y = f(x)g(x) follow the steps of the logarithmic differentiation below.

1. Take logarithm of both sides of the equation $y = f(x)^{g(x)}$:
2. Rewrite the right side $\log f(x)^{g(x)}$ as *g(x). log(f(x))*:
3. Differentiate both sides.
4. Solve the resulting equation for $\frac{dy}{dx}$.

Based on the above information answer the following questions:

1 If $y = x^{2x}$, then $\frac{dy}{dx} =$

(A) $2y(1+\log x)$

(B) $x^{2x}(1+\log x)$

(C) $x^{2x}(x+\log x)$

(D) $2x^{2x}(x+\log x)$

2 If $y^x = e^{y-x}$, then $\frac{dy}{dx} =$

(A) $\frac{1+\log y}{y\log y}$

(B) $\frac{(1+\log y)^2}{y\log y}$

(C) $\frac{(1+\log y)}{(\log y)^2}$

(D) $\frac{(1+\log y)^2}{\log y}$

3 If $y = (x+1)^{\cot x}$, then $\frac{dy}{dx} =$

(A) $(1+x)^{\cot x}\left[\frac{\cot x}{1+x} - \cos ec^2 x.\log(1+x)\right]$

(B) $(1+x)^{\cot x}\left[\frac{\cot x}{1+x} + \cos ec^2 x.\log(1+x)\right]$

(C) $\left[\frac{\cot x}{1+x} - \cos ec^2 x.\log(1+x)\right]$

(D) $(1+x)^{\cot x}\left[\frac{\cot x}{1+x} - \cos ecx \cot x.\log(1+x)\right]$

Q.NO.18. An ant is walking along a path which is given by the function $(t) = at + bt^2$, where f(t) represents the distance it covers in cm from the starting point in time ‘t’(measured in seconds).

Based on this information answer the following :

1. f(t) is :

i) a constant function

ii) a continuous linear function

iii) a continuous polynomial function

iv) a discontinuous function

2. $\frac{d}{dt}f(t)$at t = 3 seconds is

i) $a + 2b$

ii) $a + 6b$

iii) $2a + b$

iv) $6a + b$

3. The distance travelled by the ant at the end of 5 seconds is

i) $a + 10b$

ii) $5a + 25b$

iii) $10a + b$

iv) $25a + 5b$

4. $f'(t)$is

i) Is a constant function

ii) Is a polynomial function

iii) Is a discontinuous function

iv) Is an exponential function

5. If $b = 0$, then $f(t) = at$, so that $\frac{f(t)}{t} = a = f'(t)$.

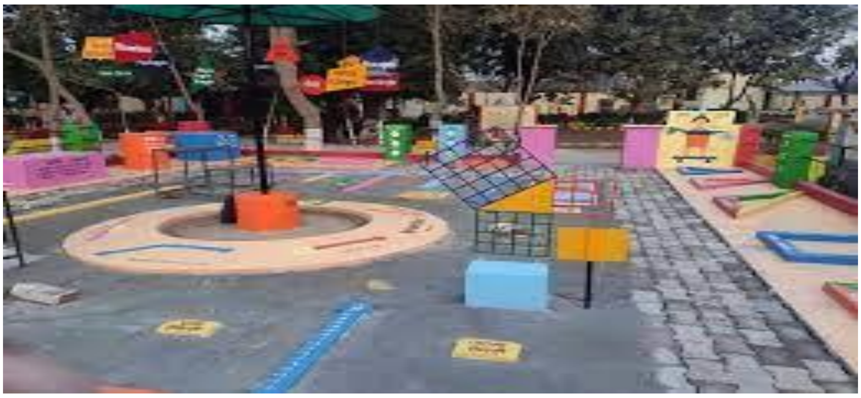
i) implies that $f'(t)$ is the constant speed $'a'$ with which the ant is walking.

ii) implies that $f'(t)$ is the constant acceleration $'a'$ with which the ant is walking.

iii) implies the ant is stationary.

iv) none of these

Q.NO. 19



MrManiratnam is a Maths teacher. He used to go to park everyday for morning walk. There he observed many mathematical figures like circle, ellipse, parabola, tetrahedron, sphere, hemisphere etc. Being a Maths teacher, after Walk , he tries to frame the mathematical equation of the figures and see the results geometrically after differentiating them.

a) If $y^2 = ax^2 + bx + c$, then $y^3 \frac{d^2y}{dx^2}$ is

i) A constant ii) A function of x only

iii) A function of y only iv) A function of x and y

b) If $y = acoslogx + bsinlogx$, then $x^2y_2 + xy_1$ is

i) 0 ii) y iii) -y iv) None of these

c) If $y = \frac{ax+b}{x^2+c}$, then $(y + 2xy_1)y_3$ is

i) $3(xy_2 + y_1)y_2$ ii) $3(xy_2 - y_1)y_2$

iii) $3(y_2 + y_1)y_2$ iv) $3(xy_2 + y)y_2$

d) If $f(x) = |x|^3$, then which one is true?

i) $f^{//}(x)$ does not exist for all real x ii) $f^{//}(x)$ exists for all real x

iii) $f^{//}(x) = 0$ iv) None of these

e) If f(x) is an odd function then $f^{/}(x)$ is also

i) Odd ii) Even iii) Neither odd nor even iv) None of these

Q.NO. 20

In the Math House, Mr Alberto used to live. Every aspect of the house is seen in Mathematical view. Mr Alberto likes to solve problem on Maths. He tried the following problems to solve.

a) Derivative of $x^{sin^{-1}x}$ $w.r.t. sin^{-1}x$ is

i) $x^{sin^{-1}x}\left(logx - \frac{\sqrt{1-x^2}}{x}sin^{-1}x\right)$ ii) $x^{sin^{-1}x}\left(xlogx + \frac{\sqrt{1-x^2}}{x}sin^{-1}x\right)$

iii) $x^{sin^{-1}x}\left(logx + \frac{\sqrt{1-x^2}}{x}sin^{-1}x\right)$ iv) $x^{sin^{-1}x}\left(xlogx - \frac{\sqrt{1-x^2}}{x}sin^{-1}x\right)$

b) If $y = sec^{-1}\left(\frac{x+1}{x-1}\right) + sin^{-1}\left(\frac{x-1}{x+1}\right)$, then $y_1$is

i) 0 ii) 1 iii) -1 iv) None of these

c) If $-\frac{\pi}{2} < x < 0$ and $y = tan^{-1}\sqrt{\frac{1-cos2x}{1+cos2x}}$, then dy/dxis

$i)$ 1 ii) -1 iii) 0 iv) 2

d) If $y = log\sqrt{tanx}$, then the value of $\frac{dy}{dx}$ at $x = \frac{\pi}{4}$ is

$i)$ 1 ii) -1 iii) 2 iv) 1/2

e) If $x = tan\left(\frac{logy}{a}\right)$, then which one is true?

i) $(1+x^2)\frac{d^2y}{dx^2} + (2x+a)\frac{dy}{dx} = 0$ ii)$(1+x^2)\frac{d^2y}{dx^2} - (2x-a)\frac{dy}{dx} = 0$

iii) $(1+x^2)\frac{d^2y}{dx^2} - (2x+a)\frac{dy}{dx} = 0$ iv) $(1+x^2)\frac{d^2y}{dx^2} + (2x-a)\frac{dy}{dx} = 0$

Q.NO. 21. A function f(x) is continuous at x = c if $\lim_{x \to c-} f(x)$ = $\lim_{x \to c+} f(x)$ = f(x) .

A function f(x) is derivable at x = c if $\lim_{x \to 0} \frac{f(c+h)-f(c)}{h}$ = f'(c)

A real valued function 'f' is finitely derivable at any point of its domain, it is necessarily continuous at that point.

Converse. A real valued function 'f' is continuous at any point of its domain, it is necessarily derivable at that point.

For example, the function f(x) = $|x|$ is continuous but derivable at x = 0.

Based on the above information, answer the following :

1. The value of 'k' so that the function 'f' : f(x) = $\begin{cases} kx+1, if\ x \le 5 \\ 3x-5\ if\ x>5 \end{cases}$

   Is continuous at x = 5 is :
   (a) 3/5
   (b) 6/5
   (c) 9/5
   (d) 12/5

2. The values of 'a' and 'b' such that the function : f(x) = $\begin{cases} 5 & if\ x \le 2 \\ ax+b & if\ 2<x<10 \\ 21 & if\ x \ge 10 \end{cases}$

   is continuous :
   (a) a=1 = b
   (b) a=2 = b
   (c) a=1, b=2
   (d) a=2, b=1

3. The relationship between 'a' and 'b' so that the function 'f' defined by :
   f(x) = $\begin{cases} ax+1, if\ x \le 3 \\ bx+3,\ if\ x>3 \end{cases}$
   is continuous at x = 3 is :
   (a) a = b + 2/3
   (b) a = b + 1/3
   (c) a = b – 2/3
   (d) a = b – 1/3

4. The values of 'a' and 'b' of the function : f(x) = $\begin{cases} ax^2+1, if\ x<1 \\ 2x+1, if\ x \ge 1 \end{cases}$

   is differentiable at x = 1 is :
   (a) a = 1, b = 2
   (b) b = 2, b =1
   (c) a = 1, b = 1
   (d) a =2, b =2

Q.NO. 22 A function f(x) is said to be continuous in an open interval (a, b), if it is continuous at every point in that interval. A function f(x) is said to be continuous in the closed interval [a, b], if f(x) is continuous in (a, b) and $\lim_{h\to 0} f(a+h) = f(a)$ and $\lim_{h\to 0} f(b-h) = f(b)$.

If function f(x) = $\begin{cases} \frac{[\sin(a+1)x+sinx]}{2} & if\ x<0 \\ c\quad if\ x=0 \\ \frac{[\sqrt{x+bx^2}-\sqrt{x}]}{bx^{\frac{3}{2}}} & if\ x>0 \end{cases}$

is continuous at x = 0.

Based on the above information, answer the following :

(1) The value of a is :
   (a) -1/2
   (b) 0
   (c) -3/2
   (d) ½
(2) The value of b :
   (a) 0
   (b) 1
   (c) -1

(d) Any real number.

(3) The value of c is :

(a) -1

(b) 1

(c) ½

(d) -1/2

(4) The value of a + c is :

(a) 0

(b) -1

(c) 1

(d) -2

(5) The value of c – a is :

(a) 0 (b) - 1 (c) 1 (d)2

ANSWERS

Q.No.1.1	(A) $\frac{1}{\sqrt{2}}$
Q.No.1.2	(B) 1
Q.No.1.3	(C) $3x^3e^{x^3}$
Q.No.1.4	(A) 2
Q.No.1.5	(A) $\frac{2}{27}x^2(2x^3+15)^3$
Q.No.2.1	(B) $\frac{-\sin(\sqrt{x})}{2\sqrt{x}}$
Q.No.2.2	(B) $-\tan t$
Q.No.2.3	(B) Differentiable at all points x∈ $R-\{0\}$
Q.No.2.4	(B) $2\text{x}\cos(x^2)$
Q.No.2.5	(A) $\frac{-\sin x}{1+\cos y}$
Q.No.3.1	A
Q.No.3.2	D
Q.No.3.3	D
Q.No.3.4	C
Q.No.3.5	D
Q.No.4.1	C
Q.No.4.2	D
Q.No.4.3	D
Q.No.4.4	B
Q.No.4.5	D
Q.NO.5.A	a
Q.NO.5.B	a
Q.NO.5.C	d
Q.NO.5.D	a
Q.NO.6.A	a
Q.NO.6.B	b
Q.NO.6.C	a
Q.NO.6.D	B
Q.NO.7.A	Iii
Q.NO.7.B	ii
Q.NO.7.C	i
Q.NO.7.D	Iii
Q.NO.7.E	ii
Q.NO.8.A	ii

Q.NO.8.B	Iv
Q.NO.8.C	iii
Q.NO.8.D	i
Q.NO.8.E	Iii
Q.NO.9.A	Option (a) $\sin x^3$
Q.NO.9.B	Option (b) $sin^3 x$
Q.NO.9.C	Option (c) $3sin^2 x \cos x$
Q.NO.9.D	Option (d) $3x^2 \cos(x^2)$
Q.NO.9.E	Option (d) - 2
Q.NO.10.A	Option (a) $\frac{1}{\sqrt{2}}$
Q.NO.10.B	Option (a) - 1
Q.NO.10.C	Option (c) $\frac{1}{3x^3 e^{x^3}}$
Q.NO.10.D	Option (a) 2
Q.NO.10.E	Option (a) $\frac{2}{27} x^2(2x^3+15)^3$

Q.No.11.	Answer
(i)	$(a)\ 0$
(ii)	$(a)\frac{1}{a}$
(iii)	$(c)\ -\frac{3}{2}$
(iv)	$(d)\ 2e^{-x} \sin x$
(v)	$(b) - n^2 y$
Q.No.12	Answer
(i)	$(a)\ y(1+\log x)$
(ii)	$(a)\frac{\log x}{(1+\log x)^2}$
(iii)	$(a)\frac{y(y-x\log y)}{x(x-y\log x)}$
(iv)	$(a)\frac{y(x-1)}{x(y+1)}$
(v)	$(c)\frac{x-y}{\text{x}\log x}$
Q. No.13	Answer
(i)	(b)
(ii)	(c)
(iii)	(d)
(iv)	(d)
Q. No.14	Answer

(i)	(c)
(ii)	(b)
(iii)	(a)
(iv)	(d)
Q.NO.15	ANSWER
1	C
2	A
3	C
4	C
5	D
Q. NO 16	
1	B
2	A
3	A
4	C

Q.NO.17	Answer
1	A
2	B
3	D

Q.NO.18	Answer
1	iii
2	ii
3	ii
4	ii
5	i
Q.19	
a	I
b	I
c	Ii
d	I
e	iv
Q.20	
a	iii

b	iii
c	i
d	ii
e	i
Q.21	
1	c
2	d
3	a
4	a
5	a
Q.22	
1	c
2	d
3	c
4	b
5	d